Analysis of Microwave Occultation Techniques for Atmospheric Soundings

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Abstract

Multi-satellite schemes to sound the Earth's atmosphere are reviewed from the standpoint of data handling, and some important problems are discussed, such as: (1) Abelian inversion of a limited number of phase measurements; (2) Exclusion of water vapor refraction; (3) Aspherical stratification effects of water vapor. Data analysis techniques for the Mars microwave occultation experiment of 1965 are examined, with special attention given to inversions of phase delay and their potential in Earth-atmosphere investigations. Difficulties inherent in techniques which do not perform an analytic data inversion along the refracted ray are considered.

Numerical estimates are made of water vapor effects upon tangent ray height and spherical stratification of refractivity. A dispersion method for retrieving air density is presented and its errors are summarized qualitatively. The differential refraction is tabulated for a satellite pair, one synchronous.

ACKNOWLEDGMENT

I. INTRODUCTION

A. Background

Successful probing of the atmosphere on Mars and Venus by radio occultation experiments has fostered hope that a similar technique might be perfected to sound the Earth's atmosphere. Accordingly, proposals have been submitted to NASA for such a system.

Meanwhile, several important papers which bear on the potential feasibility of such a technique have been recently published. There is a rather sharp discordance upon the merits of an Earth microwave occultation system, partly because its analysis is complex. A great deal of the complexity is well understood by our group which has been studying occultation - refraction theory for several years in the development of an optical refraction technique for Earth orbit.

Since many of the analyses which are completed and some basic computer programs which are available from the optical technique are applicable either directly or with modification to the microwave studies, we have participated in the analysis and assisted both the experimenters, by criticism and recommendations, and NASA, by membership on the Microwave Occultation Study Group.

B. Objectives.

The project objectives were: to examine the proposed methods critically in order to determine all problem areas; to point out the analyses required to evaluate the severity of a problem; to analyze those problems for which we had special competence; to recommend solutions based on the analyses or indicate lack of same. Particular emphasis was to be placed on the interpretation of data or inversion techniques because of considerable experience in that area. Specific studies were required on the conversion of refractivity to density in the presence of water vapor, and the effect of water vapor on other aspects of the method.

II. ANALYSIS OF CURRENT MICROWAVE TECHNIQUES

The microwave techniques to be analyzed are: the planetary fly-bys (primarily Mariner IV),the proposals of two and six satellites in low Earth orbit, the suggestions of Tatarskiy, and one-low, one-synchronous satellite pair.

The basis of all microwave techniques is the propagation of 1 to 5 Ghz radiation tangentially through the atmosphere. The index of refraction is a strong function of the neutral density, and causes the radiation to be slowed and to follow a refracted curved path. If the propagation is between suitably instrumented spacecraft the change in phase path length can be detected with extreme precision. By inference these phase changes can be related to refractivity and then to atmospheric parameters. By rapidly varying the height of the path in the atmosphere, as in an occultation,or by sampling along several heights simultaneously, a vertical atmospheric profile may be obtained.

A. Mars Occultation Experiment

1. Background

The occultation method of determining the structure of a neutral planetary atmosphere was first attempted by the single-frequency bistatic radar-occultation experiment on the Mariner IV-Mars fly-by in 1965. This experiment implemented the theory expounded in a doctoral dissertation [Fjeldbo 1964]. The application of the theory to the study of planetary neutral atmospheres and to the ionosphere with specific reference to the Mariner-type equipment was given in two papers by Fjeldbo and Eshleman [1965,a,b]. Following the successful Mariner IV-Mars fly-by, the data were treated in a preliminary fashion by the experimenters [Kliore, et al. 1965] and in revised form [Fjeldbo, et al. 1966 a,b].

Subsequently, the reduction of Mariner IV data by either approximate inversion or model-fitting methods was questioned in light of the available exact inversion solution [Phinney and Anderson 1968]. To be exact, this solution requires data not available from the single-frequency experiment, although it may be

closely approximated. Shortly afterward, the Mariner IV data was criticized more severely in a letter [Harrington, et al. 1968] showing that the published results were not unique. This non-uniqueness derived specifically from the model-matching technique. A further, and perhaps final, analysis of the Mariner IV data was presented by the experimenters which was stated to be a result of integral inversion [Fjeldbo and Eshleman 1968]. It was not, however, the exact inversion that was utilized. Hays and Roble [1968] have analyzed the exact inversion and shown its generality in retrieving several parameters, including the index of refraction. Their analysis, however, unlike that of Phinney and Anderson is not addressed to the Mariner IV data and they have not recommended a technique for data reduction. Indeed, their analysis shows that an exact integral inversion is not possible with single-frequency data because the phase-delay, as a function of position or time, is not equivalent to phase-delay data as a function of the ray-path-constant. The possibility of an iterative method to treat data analysis by the equation given by Hays and Roble, or by the recommendation of Phinney and Anderson, apparently has been rejected by the experimenters. This apparent rejection is justified by qualitative remarks in the referenced paper [Fjeldbo and Eshleman 1968]. These remarks, while convincing, are not supported by numerical analysis and thus the question will be considered below.

There are other papers which are important to reference but do not add significantly to the analysis of Mars' occultation data. Sodek [1968] discusses a Jovian experiment and Kliore et al. [1968] the Venus-Mariner V results. Harrington and Grossi [Raytheon 1967, 1968] have studied a two-satellite technique for Mars and have submitted this to NASA in several technical reports. Their data treatment is similar to Fjeldbo's and will be mentioned in the analysis.

2. <u>Analysis</u>

Because this survey does not address itself to the accuracy of the Mariner IV results, no attempt will be made to judge the merits of the varying points of view as they apply to Mars. However, the analytic techniques and their

fundamental merits and differences are important to the analysis of how the Earth's atmosphere might be investigated by a radar-occultation technique. Therefore, the analytic techniques will be compared and discussed relative to their potential significance on Earth-atmosphere methods.

A certain dissimilarity in symbolism exists between the various authors. In order for a comparison of techniques we shall rewrite the material in terms of common symbols. These symbols will be consistent with those used elsewhere in this report.

r	radius from the planet's center
ρ	atmospheric density
μ	index of refraction
N	refractivity $(\mu-1) \times 10^6$
θ	central angle
Ω	central angle
ϕ	total phase (cycles)
Φ	phase shift along a ray
f	frequency (hz)
R	angle of ray rotation
L	geometric distance between satellites
z	zenith angle; angle between planetary radius and ray
k	Dale and Gladstone constant; $\mu = 1 + k\rho$
$\dfrac{k*}{2}$	Lorenz-Lorentz constant; $\mu^2 = 1 + k*\rho; \dfrac{k*}{2} \cong k$
η	μr

$$\left.\begin{array}{l} \mu r \sin z \\[4pt] \eta_o \end{array}\right\} \left\{\begin{array}{l} \text{impact parameter} \\[4pt] \text{ray-path constant} \end{array}\right.$$

ψ	angle between spacecraft trajectory and ray path reception
α	angle between ray path reception and straight line to source

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r_m any arbitrary radius where $\mu(r_m) - 1 = \epsilon$ and ϵ may be considered zero for data processing

subscripts

o   tangent point on a ray, $\sin z_o = 1$ (r_o, μ_o, η_o)

s   conditions at a spacecraft

m   an arbitrary point at which $\mu = 1$

e   condition assuming no atmosphere

When a microwave signal is transmitted from a spacecraft in the vicinity of a planet to a receiver on earth, it is possible by proper instrumentation and tracking to detect and record the signal frequency as a function of time as well as the exact trajectory of the spacecraft under the influence of initial velocity and the planetary gravitation. After removing the Doppler shift due to the relative velocity of the spacecraft and Earth receiver, the remaining Doppler shift may be assumed due to the influence of the planetary neutral atmosphere and ionosphere, which will have a refractive index other than unity. The refractive index may likewise be expected to exhibit a marked gradient along any radius. In addition, the refractive index may have horizontal gradients, that is, gradients along parallels and meridians at constant height.

Reference to Figure 1, adapted from Phinney and Anderson [1968], may be helpful. Hays and Roble [1968] state that the phase shift along the ray is

$$\Phi(\eta_o) = 2 \int_{\eta_o}^{r_m} \frac{(1-\mu)\,\eta\,\frac{dr}{d\eta}\,d\eta}{\sqrt{\eta^2 - \eta_o^2}}$$

and give the exact inversion

$$1 - \mu = \frac{1}{\pi}\frac{d\eta}{dr}\int_{\eta}^{r_m} \frac{\frac{d\left[\Phi(\eta_o)\right]}{d\eta_o}\,d\eta_o}{\sqrt{\eta_o^2 - \eta^2}}$$

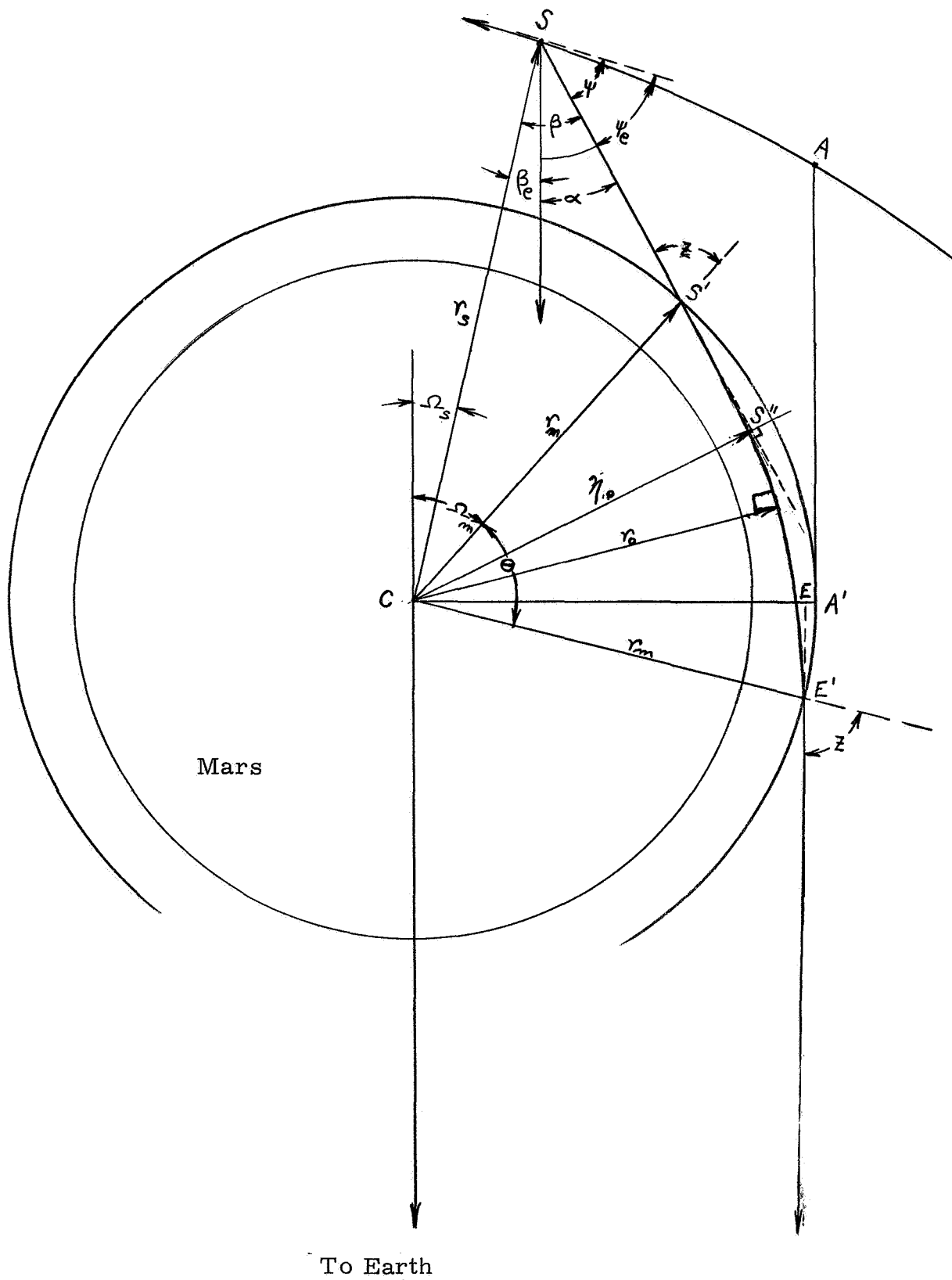They do not remark on how $\Phi(\eta_o)$ might be susceptible of measurement, or

Fig. 1 Geometry of microwave occultation experiment.

the adaptability of their inversion to computation.

Phinney and Anderson [1968] have treated these problems extensively. Their statement of phase shift is consistent:

$$\phi(\eta_o) = 2 \frac{f}{c} \int_{\eta_o}^{r_m} \frac{\mu \eta \frac{dr}{d\eta} d\eta}{\sqrt{\eta^2 - \eta_o^2}}$$

as is their inversion

$$\eta^2 \frac{d(\ln r)}{d\eta} = \frac{d}{d\eta} \left[\frac{c}{\pi f} \int_{r_m}^{\eta} \eta_o \phi(\eta_o) (\eta_o^2 - \eta^2)^{-1/2} d\eta_o \right]$$

They then integrate the inversion and introduce an $\cosh^{-1}$ function to simplify:

$$r(\eta) = r_m \exp\left[-\frac{c}{\pi f} \int_{r_m}^{\eta} \cosh^{-1}(\frac{\eta_o}{\eta}) \frac{1}{\eta_o} \frac{d\phi}{d\eta_o} d\eta_o \right] \ .$$

They have also inverted the expression for the central angle θ:

$$\theta(\eta_o) = 2 \int_{\eta_o}^{r_m} \frac{\eta_o}{r} (\eta^2 - \eta_o^2)^{-1/2} \frac{dr}{d\eta} d\eta$$

$$\ln\left[\frac{r(\eta)}{r_m}\right] = \frac{1}{\pi} \int_{r_m}^{\eta} \frac{\theta(\eta_o) d(\eta_o)}{(\eta_o^2 - \eta^2)^{1/2}}$$

Having thus demonstrated an exact analytic inversion in terms of phase shift as a function of the impact parameter, their task is to show how the data, either $\phi(\eta_o)$ or $\theta(\eta_o)$, can be extracted from Doppler shift as a function of time.

The purpose of their paper was to point out that the Mars experimenters had used an approximate method and to give a more general method. The crux of the problem is that the ray impact parameter η_o, or the ray-path-constant, is not readily available to associate with the measured ϕ. From Phinney and Anderson:

"We now show how an impact parameter η_o is associated with each ray, permitting use of one of the inversion formulas for the refractive index. Fjeldbo and Eshleman describe an approximation to determine the angle of the ray. This information is contained without ambiguity in the following way."

Their method consists of correcting the Doppler shift for relative motion of earth receiver and planet center. Having converted the Doppler data to planetocentric coordinates, they then select a time for phase zeroing. This is taken with the spacecraft at A when the ray path is above the atmosphere at an arbitrary $r = r_m$. Next they note $(\nabla \phi)^2 = \mu^2 f^2 / c^2$

$$|\nabla \phi| = \mu f/c \text{ and } \mu = 1 \text{ at the spacecraft}$$

$$\therefore |\nabla \phi| = f/c$$

Then they "determine the directional derivative $\dfrac{\partial \phi}{\partial s}$ along the trajectory from the Doppler data. The angle between the ray and the trajectory then becomes

$$\cos \psi = \frac{\partial \phi}{\partial s} \Big/ |\nabla \phi| = \frac{c}{f} \frac{\partial \phi}{\partial s}$$

where f may without error be taken as constant during the occultation. Since $\beta + \psi$ is known, β follows immediately."

They continue by considering an unperturbed ray from the spacecraft, and changing the independent variable to time:

$$\frac{\partial \phi}{\partial s} = \frac{1}{\dot{s}} \frac{d\phi}{dt} = \frac{1}{\dot{s}} \left(\frac{d\phi_e}{dt} + \frac{d\phi_{atm}}{dt} \right)$$

The second term is the portion of the Doppler shift attributable to the atmosphere.

They then have

$$\cos \psi = \frac{c}{\dot{f}s} \frac{d\phi}{dt} = \frac{c}{\dot{f}s} D$$

and

$$\cos \psi_e = \frac{c}{\dot{f}s} \frac{d\phi_e}{dt} = \frac{c}{\dot{f}s} D_e$$

so that the refraction angle

$$\alpha = \psi_e - \psi = 2 \arctan \frac{D_{atm}}{\left[D_s^2 - (D_e + D_{atm})^2\right]^{1/2} + \left[D_s^2 - D_e^2\right]^{1/2}}$$

where $D_{atm} = \frac{d\phi_{atm}}{dt}$, $D_s \frac{\dot{f}s}{c}$, and $D_s \geq D_e$

Since $\eta_o = r_s \sin \beta = r_s \sin (\beta_e + \alpha)$,

η_o is given unambiguously by $\frac{d\phi}{dt}$.

In analyzing this derivation it appears that ϕ is a function of only s before the ray path enters the atmosphere and $\frac{\partial \phi}{\partial s}$ is indeed the directional derivative. It might well be called $\frac{d\phi}{ds}$. However, once the ray path enters the atmosphere $\frac{\partial \phi}{\partial s}$ is not the directional derivative and $\cos \psi \neq \frac{c}{f} \frac{\partial \phi}{\partial s}$, due to the fact that $\frac{d\phi_{atm}}{dt}$ is not caused by bending of the ray alone but also by the retardation while the refractive index is not unity. This statement is equivalent to the statement that phase shift data lacking the impact parameter is insufficient analytically to imply refraction angles. If the impact parameter were known in addition, then the exact inversion could be employed, $\mu(r)$ determined, and the refraction angles calculated. The point is subtle, but important in our application to the Earth techniques where because of water vapor, abrupt refractive index variations can occur and the impact parameter may not be a smooth and monotonic function of time.

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Phinney and Anderson proceeded from this derivation of an "unambiguous impact parameter" to the conclusion that simplicity in computations made certain approximations desirable. The first approximation is on the impact parameter (!) which on Mars has an effect of only one digit in the eighth place. This corresponds to a phase delay error which is overwhelmed by the Doppler noise. The most significant approximation is in equation (46) where the impact parameter η_o in the denominator is replaced by a constant "mean radius" and moved in front of the integral sign. Some remarks are given as to the possible consequences but a numerical analysis is not attempted.

Summarizing the comments on Phinney and Anderson, they have demonstrated the exact analytic inversion of the phase delay as a function of the impact parameter. They have not demonstrated an exact method for obtaining the impact parameter from Doppler or total phase measurements. The error is small on Mars but not necessarily so on Earth. They have suggested approximations to assist in calculations which are not "necessary" in a mathematical sense.

Kliore, Fjeldbo, Cain, Eshleman and others involved in the Mariner IV experiment utilized model-matching techniques in the initial data treatment. The tremendous interest in a discovery experiment certainly justified the publication of preliminary results, whether or not processed by the most sophisticated method. The unknown composition plus the significant ionospheric perturbation left many parameters to be determined from a small amount of data. The Doppler residuals contained a large amount of noise. All-in-all the model-matching gave a preliminary estimate, which was all that was claimed. [Kliore, et. al. 1965] . The most important objection to the published results in 1966 and 1967 was voiced by Harrington, Grossi and Langworthy [1968] by their clear demonstration of non-uniqueness. With the final data treatment of Fjeldbo and Eshleman [1968] we have probably the ultimate which can be derived. Certain assumptions on composition must still be made. After this, the data is subjected to integral inversion. However, the inversion is approximate because only a straight ray

was considered. Fjeldbo [1964] applied the Abel inversion to the straight-ray equation

$$\Phi (r) = \frac{2f}{.3} \int_{r_o}^{\infty} \frac{[1- \mu \ (r)] r \ dr}{\sqrt{r^2 - r_o^2}}$$

He evidently did not try to integrate along the refracted ray path. One might assume that the title of Fjeldbo and Eshleman 1968 "... Integral Inversion of Mariner IV Occultation Data" was in response to Phinney and Anderson's recommendation. (The latter acknowledged the comments of the experimenters on their MS.) However, it clearly is not, as the suggestion is rejected and the straight ray is inverted. The causes for rejection are important:

"There are several techniques that may be utilized to compute N(r) from $\phi_1(r_o)$ [Fjeldbo, 1964]. The method to be outlined here was chosen because it does not require any pre-inversion filtering of the raw doppler residual data. This approach may offer three advantages: (1) the refractivity profiles are available in a form where the altitude resolution is unaffected by any particular filter characteristics (other than the 12 hz phase-locked-loop bandwidth); (2) post-inversion filtering may require less computer time since it is unnecessary to repeat the inversion computation if one wants to change the filter; (3) the physical properties of the atmosphere are related more simply to the parameters of a matched filter applied to N(h) than to the doppler data."

In examining these causes we note that: (1) Fjeldbo's "several techniques" did not include an Abelian inversion along the refracted ray; (2) in as important and expensive an experiment as Mariner IV it is surprising that computer time is a parameter; (3) certainly a smoothing is required of the doppler residuals for an Abelian inversion, since Φ must be differentiable and continuous; it is not clear how noise of 3-4 km period permits finer vertical resolution if removed after inversion, and (4) cause (3) is not understood. The gist of their argument and the justification for straight-ray approximations is contained in the appendices: the

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errors inherent in the measurement technique far outweigh those resulting from integral approximations in data processes; therefore elaborate effort to determine the impact parameter (for example) is uncalled for.

In their report on the feasibility of a Martian orbiting pair, Harrington and Grossi [Raytheon 1967, 1968] develop the same Abelian inversion of the straight-ray as did Fjeldbo. Their report notes the errors from such approximations as well as those by model matching, assuming mild and severe horizontal gradients, and straight rays vs. appreciable bending. The results are noteworthy in their application to an Earth-orbiting pair. The Abelian inversion of a straight ray, when the actual path was "appreciably" (?) curved (as it would be on Earth) was intolerably erroneous. The Abelian inversion was applied in one case to a discontinuous phase-delay function, which is interesting because it does not satisfy the mathematical restrictions on Abelian inversions. This restriction is discussed further in the appended section on the Abel integral. The results of their error analysis are applied in the section on Earth atmosphere techniques. A comparison of their error results with those given by Fjeldbo and Eshleman [1968] will not be done because it applied only to the validity of the Martian results.

B. Earth-atmosphere technique

1. Background

The Earth-atmosphere techniques were given original impetus by a student group at Stanford University which proposed a system called SPINMAP [Stanford, 1966] . The test of a similar system was proposed by the Stanford Electronics Laboratory to be carried on Nimbus E Meteorological Satellite [Stanford, 1968] . Although this system is not designed as an operational data-gatherer, it would if implemented provide a test for the equipment design and a feeling for the quality of raw phase data to be expected. Since no publication of the general technique exists in the literature, many of the details were furnished by private communications with the principal investigator, B. B. Lusignan.

An occultation technique consisting of two orbiting satellites was proposed for ionospheric measurements [Harrington and Grossi 1968] . Some features are similar. A Soviet scientist, V. I. Tatarskiy, has also analyzed a two-satellite method in Earth orbit gathering microwave or optical data, or both. Reporting results in two recent papers, his conclusions are of considerable interest. Primary attention will be paid to the Stanford proposal, even though unpublished, since it is most current and the subject of present study by the NASA Study Group.

2. Method

One of the chief difficulties in analyzing critically the microwave occultation technique applied to Earth is the lack of a concrete and non-varying embodiment. Several changes of critical importance have been introduced since the original proposal and after certain suggestions or criticisms new changes will probably be forthcoming. A definite attempt was made, therefore, to avoid criticism of details susceptible to change and to address problems fundamental to the basic technique.

This basic embodiment (Stanford) has one master and five slave satellites in low circular orbit. (One slave has been proposed as a test.) The satellites are launched by the same vehicle and hence begin their function in identical orbits. Equipment is carried for measuring the doppler phase shift of two-way transmission, master-slave-master, and the total phase path in cycles. The satellites are earth stabilized along a radius and control systems for command thrusting are carried. The slave satellites are thrusted slowly away from the master in a manner to keep them as nearly as possible in identical orbits. For a while the path between satellites is above the atmosphere and the doppler phase shift is a precise indication of the relative velocity. As the slave satellite approaches its desired spacing its propagation path sinks into the atmosphere and the doppler signal becomes a combination of relative velocity and atmospheric effects. When in proper position, microthrusters are employed to halt the

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relative velocity as nearly as possible. A group will be transmitted at 3Mhz so that if temporary loss of lock or loss of signal occurs the total phase path can be reestablished. Doppler shift is recorded continuously with a precision of one cycle; thus at 6 cm wavelength on a two-way path, 3 cm accuracy is possible in determining the phase path changes.

The present data process technique envisioned is the comparison of the five integrated doppler shifts at five altitudes with a predicted phase profile for certain specified atmospheric conditions, ie. model-matching.

The status of the equipment is unknown except that it is quite similar to that used on the Mariner spacecraft. Problems of launch vehicle, attitude control, microthrusting, data acquisition, recording, and playback will not become a part of this analysis since we have no special knowledge of the microwave equipment or vehicle.

The status of data handling and error analysis has been examined critically at Stanford. The presently available programs are able to predict the correct phase path length in a particular atmosphere when the radii and separation angle of the satellites are input. This is accomplished by an iterative solution whereby a sample ray is traced through the given atmosphere to see if it strikes the second satellite. If not, the angle of emission from the first satellite is adjusted and another iteration is made. The process continues until the ray is within a small miss distance of the second satellite. Since an arbitrary atmosphere may be entered into the program, horizontal gradients may be analyzed. The program is expensive, as small steps are required in the ray path integration and several trials must usually be made. That process yields one altitude level for one atmospheric model. The computer cost for a definitive error analysis would be excessive.

Their retrieval program is not far along. The atmosphere to be retrieved must be specified by only five parameters, otherwise,with five data, no convergence would be possible. With a recursive method, the phase delays are adjusted until a best fit for the data is obtained. Refractivity is assumed proportional to density.

Spherical stratification has been examined at Stanford by the technique of assuming a seemingly large pressure gradient from satellite to satellite. (No simulation of water vapor effects is possible except for the alteration of neutral density in some manner.) The first trial (and only reported) was with a separation of 0.6 radians and 650 mb at the surface under one satellite and 1300 mb at the surface under the other. The temperature was from a segmented standard atmosphere, assumed spherically-stratified. Density was calculated and the ray traced. The phase delay error was found to be negligible and an order of magnitude below our published results for non-sphericity.

If the refractivity shows a constant bias it will be attributed to residual relative velocity and subtracted. They assign no residual error to this procedure. Air drag and solar pressure were calculated and found negligible. Cross-path gradients were found to contribute negligible error.

The question of reflections in the atmosphere, ducting, multi-paths, and diffraction were studied. The only serious problem seemed to be with multi-path transmission which occured when actual water vapor data was taken at 3 km altitude, extrapolated to 5 km, then assumed to dwindle to zero over a kilometer or two. The conclusion reached was that the decrease in water vapor with height was unrealistically large and that multi-paths probably would not occur.

3. Analysis

Specific comments on instrument design will not be made. It will be assumed that the doppler phase shift and group path measurements can be made and recorded on a master satellite to the accuracies claimed.

The main problem with the multi-satellite occultation method is that there are no occultations, at least none sufficiently rapid to give a vertical function of refractivity. The multi-satellite method simulates an occultation by transmitting over several vertically separated paths. Although the transmissions are not simultaneous in the sense of having tangent points along a single radius, they may be adjusted in time so that the horizontal departures are insignificant. Since all

the published analytic inversion techniques require the input of a vertical function
of doppler or phase path, when only a limited number of measurements are avail-
able the interpolation of the function gives rise to substantial error. The more
satellites, the less interpolation error. Previous studies on refraction angle found
the interpolation error to be substantial only when the measurements were separat-
ed vertically by at least 5 km. To obtain the same accuracy in the recovery of the
refractivity - height function from microwave measurements requires seven
satellites (one master - six slave) to cover the region 5 km to 30 km. The fact
that in the optical case bending is alone considered, while in microwaves the bend-
ing and retardation are the joint cause of the perturbation, should not force a radi-
cal change in the error analysis. This is illustrated by the analogy of Hays
and Roble [1968] .

     After we pointed this out in our COSPAR paper [Fischbach, et al. 1968] ,
Lusignan proposed to invert strictly by model-matching. We do not regard the
technique favorably because it is impossible to draw definitive (analytic) error
conclusions, nor is it reasonable to assume that five measurements of phase
path can specify five meteorological parameters of another type without substan-
tial error. This is provable in the abstract sense because the density, temperature,
and pressure are analytically deducible from the phase measurements. Errors
inherent to the technique cannot possibly be removed by model-matching unless
substantial a priori knowledge of the atmosphere is available. In density and
pressure, no such help is forthcoming because the interpolation of phase
measurements will be done exponentially and improvements are difficult to
imagine. In temperature, a basic structure of lapse rate capped by isothermality
in mid-latitudes is reasonable, but one must necessarily improve on the a priori
knowledge a great amount: otherwise little reason exists for the experiment.
Lusignan makes the point that a priori knowledge is easy to apply when model-
matching and not otherwise. Since one method (Abel inversion) looks difficult but
promises definitive error analysis, and the other (model-matching) is easily done

but says little about errors, effort should be focussed on this extremely vital area.

All of the foregoing assumed that refractivity and density are related by a simple function such as $\mu - 1 = k\rho$. The problem of water vapor contribution to the index of refraction introduces an important complication. Because the water vapor molecule is polar while other constituents of the atmosphere are not, its mixing ratio is of extreme importance. This polar molecule will cause 16 times the microwave frequency refraction of a non-polar molecule of the same weight. In the troposphere, water vapor is an important constituent. On the average the mixing ratio decreases with height sufficiently that above 300 mb it can be assumed to cause negligible effect, but below that altitude its occurence is variable in the extreme. Below 800 mb it is so great that the water vapor concentration effectively masks all other atmospheric variations of the micro-wave refractivity. Between 800 mb and 300 mb altitudes, Lusignan proposes to remove the effect by assuming certain water vapor concentrations on the basis either of climatology or of adjunct measurements. One such adjunct measurement is the fluctuation of microwave signal strength, or the fluctuation of refractivity itself. Such a procedure carries a probability of serious error. If the climatological profile of water vapor is based on a Gaussian distribution, the error could be analytically determined, but no data exist to support such a Gaussian distribution. If the refractivity fluctuations as a function of horizontal motion are to indicate the proportion of water vapor on the propagation path, the theory must be presented by the proponents.

Another serious problem with water vapor predictions and the effect on inversion procedures is the assumption of spherical stratification. All analytic (and most model-matching) procedures rest on this assumption. That the assumption is valid for neutral density has been shown [Fischbach, et al. 1962] . Water vapor is not approximately spherically stratified and the resulting prob-. able error has been investigated. The preliminary result of Lusignan has assumed

a linear horizontal gradient of pressure and density between satellites. The first derivative is negligible, as our analysis has shown. Lusignan's assumption is therefore totally unrealistic.

The question of removing relative motion between master and slave satellites from the total phase or doppler measurements has been the subject of considerable discussion and will be the main topic of the next Study Group meeting. We do not claim competence on the subject of how accurately micro-thrusters can stablize one satellite with respect to another, but the fact that this residual motion is crucial to the data analysis is undeniable. All of our computations are based on the assumption that the residual motion can be removed from the phase measurements with no error.

Air drag, solar pressure, and cross-path gradients are believed to be negligible.

Diffraction, Fresnel stretch, and attenuation are not considered to be serious beam propagation problems. However, multi-path transmissions are expected. Whether the equipment can sort these out is a problem we have not addressed. It is appropriate that the proponents analyze this problem.

The suggested method of Harrington and Grossi [1968] applies to the ionosphere and atmosphere. As far as their neutral atmosphere determination is concerned, they claim 5% density accuracy with 1 km vertical resolution and 1- to 30-day time resolution. It is difficult to correlate this prospect with the synoptic requirements. Their problems are exactly those discussed above: no occultations and water vapor.

Tatarskiy [1968 a, b] has quite independently examined the theory from either an optical or microwave embodiment or both, but always from a source-sink orbiting pair. (He has thus not considered stellar-refraction, a method which alleviates some of the major problems.) His conclusions for microwave methods are that they:

1) Cannot be used below 10 km because of water vapor.

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2) Require the accurate determination of satellite coordinates (recommends error <16 cm between satellites).

3) Demand an accuracy of frequency measurements of the order of 10^{-10} for periods of 0.1 sec.

His conclusions concerning optical methods are somewhat more optimistic.

It ought to be noted that while Tatarskiy states that the satellite coordinate determination requirement is "absolutely unrealistic" he is not aware of Lusignan's novel scheme for hopefully alleviating this problem. His water vapor conclusions coincide almost exactly with our calculations.

In summarizing, we cannot see an effective inversion procedure using less than eight satellites when there are no rapid occultations. Water vapor corrections must be made below 300 mb, and if somehow made, will be subject to a sizable asphericity error. The orbit determination is always crucial - a convincing exposition of a novel method is required. Multi-path transmission must be coped with by the experimental technique.

That the method was effective on Mars is insufficient to show efficacy on Earth, because:

1) An actual occultation occurred.

2) No water vapor or polar constituents were present.

3) Required accuracy was an order of magnitude less.

One obvious deficiency exists in the technique - a definite statement of the inversion procedure. After such a method is clearly presented, an error analysis will be possible connecting beam statistics, number of satellites, orbit and phase error, and water vapor error to the errors in atmospheric parameters.

III. NUMERICAL ANALYSIS OF CERTAIN PROBLEMS

Despite the lack of exposition of a specific program for data treatment by the proponents of microwave occultation Earth applications [Stanford 1968, Raytheon 1968] , it is possible to draw some preliminary conclusions based on the phase shift effect which would be caused by hypothetical situations. Utilizing this approach we have sought to define the altitudes at which water vapor negates the determination of density, due both to unpredictable occurrence and its aspherical distribution. Also examined were certain differential refraction effects and the potential benefits of dispersion techniques.

The effect of water vapor in particular was estimated by the processing of a large quantity of actual data. The spherical stratification analysis shows why an assumed linear gradient will not produce any marked non-spherical effect. The magnitude of tolerable errors is of course important in feasibility studies but is not required for the actual analysis. We have found Sawyer [1962] to be useful in providing standards for synoptic networks. That analysis places the maximum allowable error in mid-latitude pressure between 0.5 and 3 mb., depending on altitude. Tatarskiy [1968b] used 1 mb as a standard based on radiosonde accuracy. It is interesting to note that our selection of 300 mb and Tatarskiy's 10 km as the upper limit of serious water vapor errors are nearly coincident, though the analyses were entirely different.

A. Water Vapor Effects on Microwave Radiation

The sensitivity of microwave radiation to water vapor is well known. Since this relationship has a bearing on data procurement, some quantitative information about water vapor effects will now be presented.

The geometry of Fig. 2 shows how moisture raises the tangent point in a dual-satellite configuration. The amount of ray elevation (δ) is determinable from water vapor mixing ratio data and the assumption of isothermality. Table 1 gives the results of such a computation for the San Juan, Miami, Buffalo, and
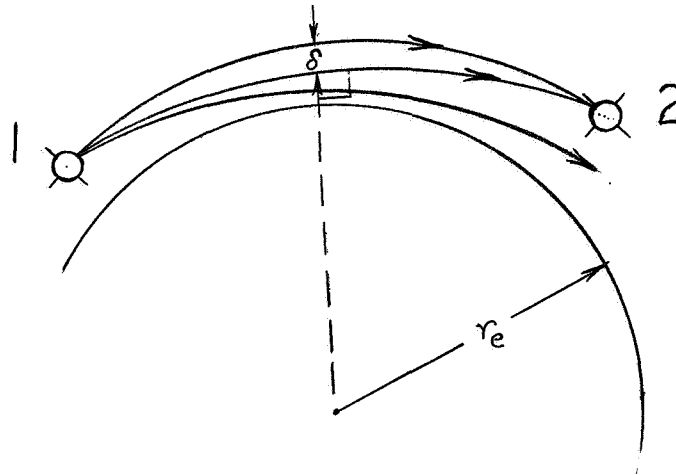
Fig. 2 The effect of water vapor on the height of the tangent point of a ray. A dry atmosphere ray traveling from satellite 1 to satellite 2 will, upon the introduction of water vapor, bend more, missing satellite 2. In the moist atmosphere, the ray which reaches satellite 2 must be elevated by a distance δ, at the tangent point. r_e = Earth's radius.

	Ground mean	σ	850 mb mean	σ	700 mb mean	σ	500 mb mean	σ
San Juan (Jan 1966)	621 m	91 m	327 m	85 m	101 m	57 m	30 m	11 m
Miami (May 1966)	645	127	321	91	115	65	36	25
Buffalo (Jan 1966)	110	58	74	55	46	32	14	10
Caribou, Me. (May 1966)	210	81	120	80	65	52	17	14

Table 1 The elevation of tangent ray height (δ) due to water vapor, assuming an isothermal atmosphere. σ = one standard deviation of the mean value.

Caribou, Me., radiosonde stations, for single months as indicated. The sizes of the standard deviations are noteworthy, and they are indicative of the variable nature of water vapor with its occasionally reversed vertical gradients. The humidity profile for Washington, D. C. on May 26, 1964 at 1424Z (Fig. 3) exemplifies the erratic behavior of water vapor which can occur.

This sensitivity of microwave refraction to water vapor may jeopardize the retrieval of atmospheric density in the following ways.

1) Unknown water vapor content along the ray path is reflected in erroneous air densities. Because of the tendency for stratification of moist layers, a measurement or an estimate of total vertical columnar water vapor content, as from satellite cloud pictures, will not suffice in the horizontal probing technique. Neither will the assignment of "Water vapor scale height" values based upon a known ground H_2O mixing ratio, although the profiles of p and T could be obtained, perhaps, in a gross sense.

2) Unknown water vapor content along the ray path introduces an error in the tangent height of the ray, or the altitude of the data point. The magnitudes of this effect were pointed out above in Table 1 for a variety of climates. This kind of error may well be harmful to the data inversion process.

3) As shown elsewhere, the assumption of "spherical stratification" is not completely fulfilled, even though it will probably be a basic assumption of a data inversion system. The resulting refraction angle error appears to be significantly large for microwave radiation under certain atmospheric conditions in the lowest 3 - 4 km.

The problem of water vapor sensitivity is alleviated, generally, at higher levels. Thus one may conceive of three layers in the troposphere:

(a) A lower layer where water vapor effects usually swamp the data inversion;

(b) An upper layer in which water vapor is seldom of any consequence;

(c) An intermediate layer in which the fluctuations of water vapor have sporadic importance. The base of the upper layer may be considered a "cut-off altitude"
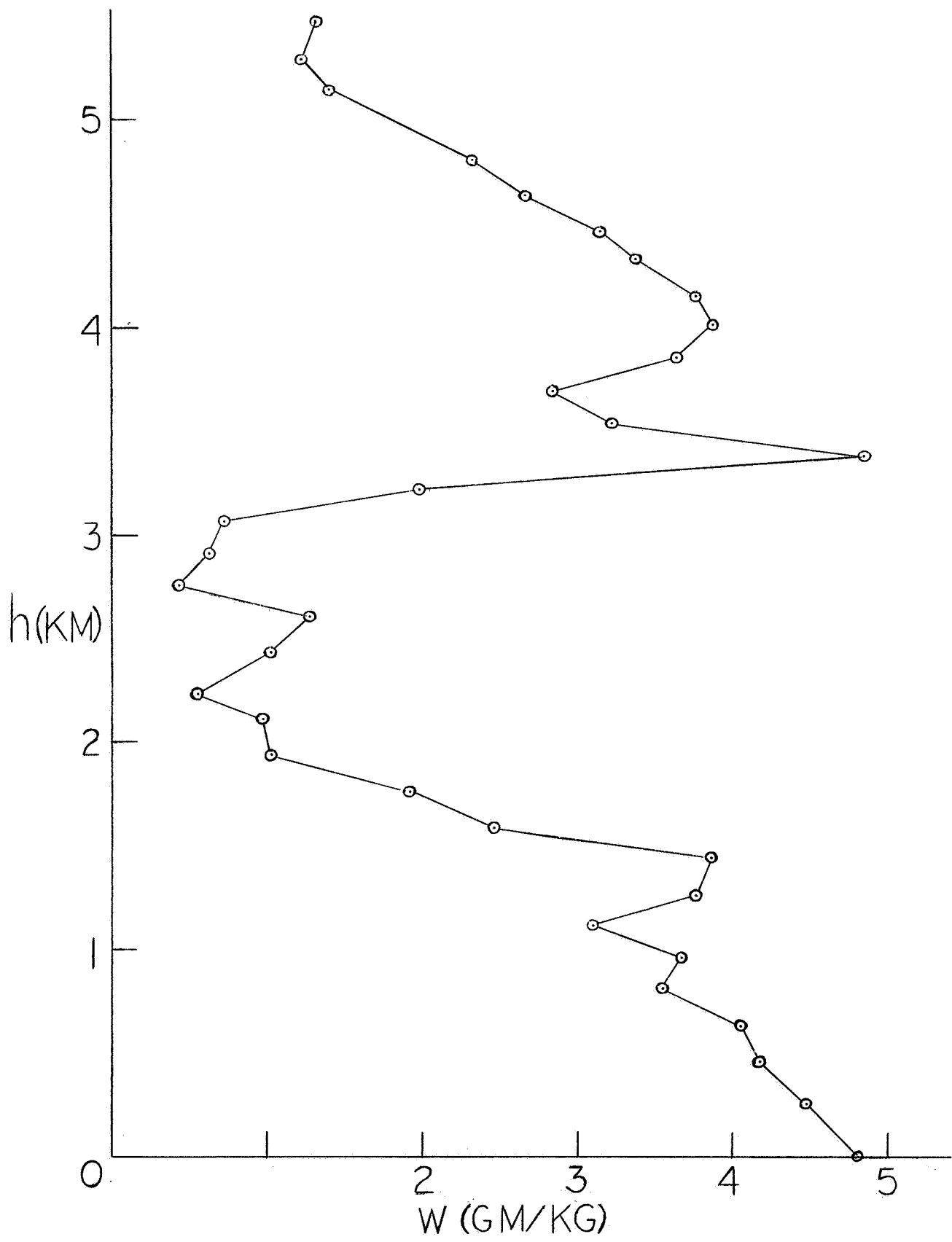
22

Fig. 3 W is the water vapor mixing ratio, h is the height.

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for water vapor, and we have selected 10 km as being perhaps the best single estimate of this altitude. Below 10 km, and outside of Intertropical Convergence Zones, cloud frequencies increase downward markedly in a broad range of latitudes [Graves, 1968] . However, under known favorable conditions the "cut-off" can be lowered half way to the ground, to 500 mb, with little error entering into the computation. Water vapor being invisible, the certain identification of such conditions from the use of satellite cloud photographs is unlikely; the information from 6.0 - 6.5 μ radiometers is more pertinent.

The sensitivity of a (polarized) water vapor molecule is known to be 16 times greater than an (unpolarized) air molecule. Therefore, statistics dealing with the mixing ratio of water vapor (q) can be utilized to determine the error resulting from the omission of water vapor refraction. The source of this information is the "Atmospheric Humidity Atlas — Northern Hemisphere" [1966] .

The data extracted from this report are 95 percentile mixing ratios in gm kg^{-1}, and they are presented in Table 2.

	500 mb (5.5 km)	400 mb (7.5 mb)
Central United States (Jan)	1.5 gm kg^{-1}	0.6 gm kg^{-1}
Central United States (July)	3.6	1.6
Central Canada (Jan)	0.7	0.2
Central Canada (July)	1.9	0.8

Table 2

The percentage error in air density due to the neglect of water vapor can be expressed approximately as

$$\epsilon' \; [\%] = 16 \times \frac{18}{29} \times q/1000 \times 100 \cong q \quad gm \; kg^{-1}$$

Thus the values in the table represent ϵ' in percent exceeded 5% of the time in the regions indicated. From previous error studies on optical refraction

[Fischbach, et al., 1965], we know that the 500 mb values are too large an error for operational meteorological purposes. When an additional error is added for aspherical stratification in the presence of water vapor, the 400 mb values are also larger than desirable in most cases. Thus, the humidity statistics support the previous conclusion drawn from vertical cloud distribution statistics. Clearly, the 500 mb and 400 mb levels lie within the intermediate, marginal layer. The "cut-off altitude" is near 300 mb, or 10 km, in the absence of supplementary water vapor information. The value of ϵ' at 300 mb is of order 10^{-1} %.

The parameters obtainable through the use of the microwave technique, i.e. temperature and pressure-height, can be extrapolated downward to 500 mb outside of tropical regions [Graves and Epstein, 1967]. This is true of any technique giving these parameters down to 300 mb, and is of course applicable to star-tracking as a source of refraction angle. The presence of clouds is an effective lower altitude limit to star-tracking, and as we have implied above, it is indicative of the lower altitude limit to operational microwave techniques unless accurate water vapor information is available.

B. Spherical Stratification

The real atmosphere departs somewhat from spherical stratification. In the case of refraction in the optical range through strong frontal zones, an analysis by Hays [Fischbach, et al. 1962] revealed negligible errors of the order of one percent of the refraction for an isothermal, spherically stratified atmosphere in the plane of the bending ray, and 10^{-1} percent laterally to the ray. The same analysis is applicable to microwave radiation, so its assumptions and approximations will be reviewed here briefly.

1. Review of Derivation of Relative Error Formula

The exact differential equation for refraction can be expressed as

$$\frac{d\vec{T}}{ds} = \frac{1}{\mu} \vec{T} \times (\vec{Grad}\,\mu \times \vec{T})$$

$\vec{T}$ = a unit vector tangent to a ray with direction s

μ = index of refraction

To evaluate the corresponding integral, a coordinate system is introduced (Fig. 4) which has x = s and y = the radius vector at the ray tangent point. Then the first iteration beyond the straight line raypath case gives

$$\frac{d\vec{T}^{(1)}}{dx} = \frac{1}{\mu} \left(\frac{\partial \mu}{\partial h} \sin \theta \, \vec{i}_y + \frac{\partial \mu}{\partial s_\theta} \cos \theta \, \vec{i}_y + \frac{\partial \mu}{\partial L} \vec{i}_z \right)$$

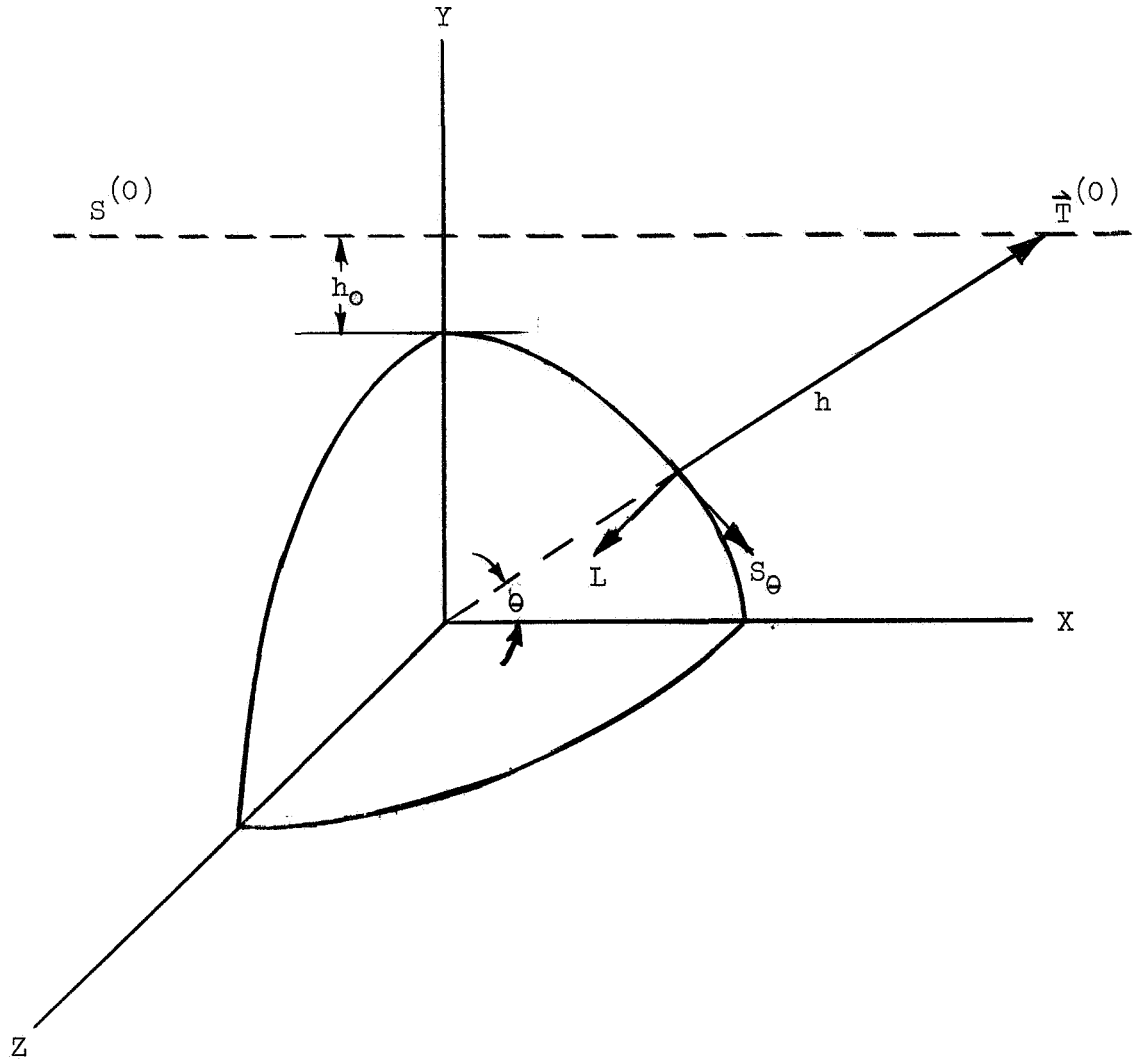where h, θ, s_θ and L are identified in Fig. 4.

Fig. 4. Coordinate system for sphericity analysis

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If the small angle assumptions are applied to $\vec{T}^{(1)}$, we get

$$\vec{T}^{(1)} = \vec{i}_x + R_p \vec{i}_y + R_L \vec{i}_z$$

Here, R_p and R_L are the desired refraction angles in and across plane xy, respectively.

A small amount of manipulation using Dale and Gladstone's Law yields

$$R_p \cong k \int_{-\infty}^{\infty} \left(\frac{\partial \rho}{\partial h} \sin \theta + \frac{\partial \rho}{\partial s_\theta} \cos \theta \right) dx$$

$$R_L = k \int_{-\infty}^{\infty} \frac{\partial \rho}{\partial L} dx$$

ρ = density

The derivatives of ρ are expressed in terms of a pseudo-exponential atmosphere, $\rho = \rho_o (s_\theta, z) e^{-(h-h_o)/H(s_\theta, z)}$

ρ_o = density at tangent point

H = scale height

After making the parabolic approximations such that

$$s_\theta \cong x, \quad h - h_o \cong \frac{x^2}{2(r_e + h_o)}, \quad \sin \theta \cong 1, \quad \cos \theta = \frac{x}{r_e + h_o},$$

r_e = Earth's radius,

the mean density and scale height are expanded along x in a Taylor's series,

$$\rho_o = \bar{\rho}_o + \bar{\rho}_o \rho'(\theta) x + \bar{\rho}_o \rho''(0) x^2/2 + \ldots + \bar{\rho}_o \rho_L(0) L + \bar{\rho} \rho'_L(0) x + \ldots$$

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where

$$\rho'(0) = \frac{1}{\rho_o} \frac{\partial \rho}{\partial x}\bigg|_{x=o} \qquad \rho_L(0) = \frac{1}{\rho_o} \frac{\partial \rho}{\partial L}\bigg|_{x=o}$$

$$\rho''(0) = \frac{1}{\rho_o} \frac{\partial^2 \rho}{\partial x^2}\bigg|_{x=o} = \frac{1}{\mu_o - 1} \frac{\partial^2 \mu}{\partial x^2}\bigg|_{x=o}$$

Integration of the approximate refraction integrals result in

$$\frac{R_p}{R_{p_o}} = 1 + \frac{H_o}{2}(r_e + h)\left[\rho''(0) + \frac{H''(0)}{2} + H'(0)\rho'(0) - 3H'(0)^2\right] + \ldots$$

$$\frac{R_L}{R_{p_o}} = -H_o\left[\rho_L(0) + \frac{H_L(0)}{2}\right]$$

These formulae give the relative errors in refraction compared to an isothermal, spherically stratified atmosphere having refraction R_{p_o}.

2. Application

The critical factor in applying these expressions to the real atmosphere is the evaluation of derivatives. In the attack upon this problem, three data sources were utilized:

1) The series of charts showing horizontal refractivity gradients for a storm system on 19 February 1952 from Bean and Dutton [1966]. These charts were scanned for large first and second derivatives of μ with respect to distance. The gradients in μ alone were sufficiently large to give a 10 percent error in R_p and 0.5 percent error in R_L at the ground, decreasing slowly with height to about 7 1/2 percent and 0.1 percent, respectively, at 700 mb (3 km). The two terms in H'(0) are

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small and tend to cancel. The term in $H''(0)$ is difficult to evaluate with-out analysis charts, but as in the case of optical refraction, it is believed to be fairly small.

2) A refractivity analysis by Martin and Wright [1963] , wherein the normal August values (N_o) in central Texas are fit by a parabolic curve along a path which nearly coincides with the gradient of N_o. The coefficients are as follows:

$$N_o = D + EX + FX^2$$

$$D = 320.4$$

$$E = 2.801 \times 10^{-5} m^{-1}$$

$$F = 1.025 \times 10^{-10} m^{-2},$$

with a standard deviation of 1.3 N_o units for the fit. The F-coefficient leads to a value of 2.88×10^{-7} km^{-2} for $\rho''(0)$, and 0.8 percent error in R_p.

3) Compilation of statistics on water vapor and scale height gradients between Pittsburgh, Pa. and Washington, D. C. (315 km) and between Columbia, Mo. and Little Rock, Ark. (460 km) at the ground, 850 mb, and 700 mb. The data supplied dependable information on the horizontal gradients of water vapor, refractivity, and scale height, but the charts referred to in Para. 1 above were needed as guidelines to higher derivatives. Physical considerations would negate finding a mean error exceeding the one percent error in R_p deduced in Para. 2 above. The maximum error in both sectors, making reasonable assumptions about the second derivatives of $\rho(0)$ and $H(0)$, is estimated to be six percent at the ground and four percent at 700 mb. The data used consisted of two soundings per day, Jan - May, 1966.

3. Conclusions

The conclusions about spherical stratification which we draw from this investigation are these:

1) The refraction angle error in a vertical plane containing the raypath is of order one percent at the ground in mean conditions across the water vapor gradient in Texas in summer. This is one of the larger mean summer-time errors in the U.S.A., because of the Gulf Coastal effect. Lateral to the ray, the error is smaller by a factor of 10.

2) Around storm systems, refractivity patterns can produce an order of magnitude increase in the two error terms. This means that the refraction angle error in the raypath plane is as much as 10 percent in the least advantageous case, and lateral error is then about 0.5 percent. These numbers pertain to ground conditions.

3) The errors decrease quite slowly with height when frontal zones are involved. At times, the refraction error in the raypath plane evidently amounts to as much as 7 1/2 percent at 700 mb (3 km) and one percent at 500 mb (5 1/2 km).

4) Spherical stratification of density appears to be a valid assumption for _mean_ conditions in most regions and at all levels, as far as microwave radiation refraction is concenred. However, pronounced water vapor gradients around certain storm centers and fronts cause a refraction angle error in the plane of the ray which is significantly large. This error is believed to exceed 5 percent at times in the first 3 km.

C. Dispersion Techniques for Retrieving Air Density

The use of two or three transmission frequencies is an attractive way to gain additional information on gases which refract and attenuate the signal. The goal in this class of operating procedures is to determine the amount of the attenuating gas along the path and to deduct its bending effect from the total refraction. An error analysis of a three-wavelength dispersion technique by Thompson [1968] , with wavelengths at 4048 $\overset{o}{A}$, 5790 $\overset{o}{A}$, and 3 cm, gave theoretical error contributions of order 1 percent for water vapor and order 10^{-1} percent for air density. Furthermore, certain refinements of the measurements hold the promise

of reducing these errors by two orders of magnitude for the ground-based approach. If the tri-frequency method could be instituted between satellites in a mother-daughter configuration, valuable water vapor information would be a possible product of the horizontal probing technique.

Since there is an inverse square relationship between refractivity and wavelength, a dual-wavelength transmission in the microwave region, as between ~ 1.35 cm and 6.00 cm, would suffer little wavelength dependence. The ray separation at the tangent point would be of the order of 10^{-2} m in this case. In the above three-frequency case, the ray separation is of order 10^{-1} m, and is still negligible.

1. Illustrative Dual-frequency Case

Since the rays in any dual-frequency system are essentially coincident, the important errors in the system must come from other sources. To illustrate the kinds of errors which may occur, consider a mother and two daughter system as in Fig. 5. The object is to find the air density at E and G, having observational evidence of phase delay $\Phi_i(r_o)$, water vapor mixing ratio, $w_i(r_o)$, and the offset point for each "straight ray", $\bar{r}_i$. The tangent ray height, $(r_o)_i$, and the refractivity, $(N_o)_i$, at $(r_o)_i$ are not known.
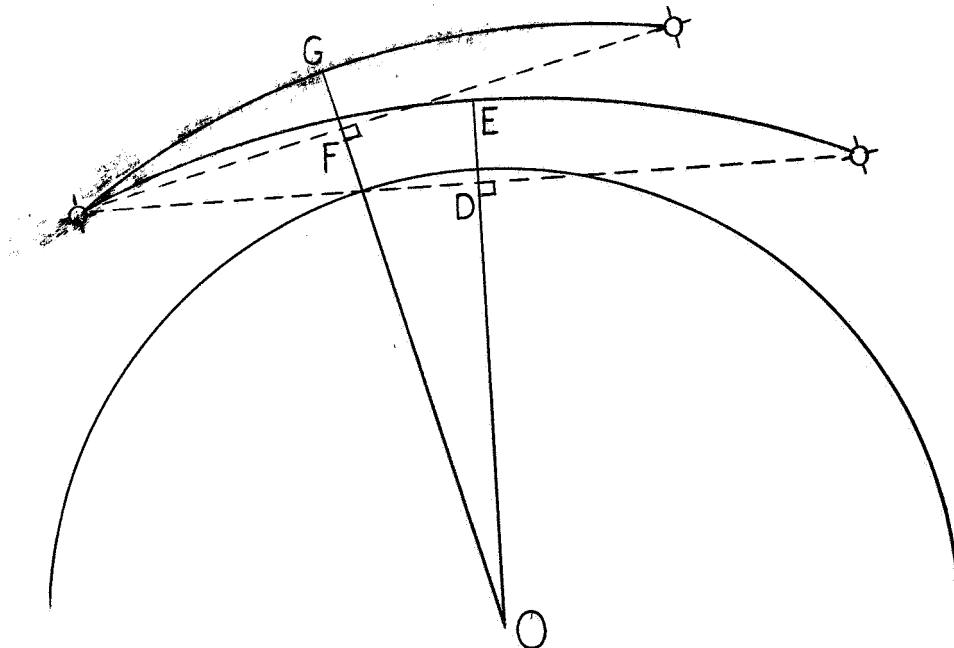

Fig. 5 Schematic drawing of tri-satellite configuration

As a simple, bootstrapping procedure, one may visualize the following sequence of approximations.

Result

1) Since $\bar{r}_D$ and $\bar{r}_F$ are known from geometry, with error, an initial estimate of N_D and N_F can be made from an appropriate standard atmosphere.

N_D

N_F

2) Use a model atmosphere to obtain estimates of r_E and r_G, water vapor being excluded. An iso-thermal model gives $\left[r_i\right]^1 = \bar{r}_i \, \mu \, (\bar{r}_i)$. a first approximation,

$\left[r_E\right]^1$

$\left[r_G\right]^1$

3) Apply the known H_2O mixing ratios at the actual r_E and r_G to $\left[r_E\right]^1$ and $\left[r_G\right]^1$.

$\left[w_E\right]^1$

$\left[w_G\right]^1$

4) From the refractivity due to water vapor, N_w, get an estimate of $(N_w)_E$ and $(N_w)_G$. Note that $N_w = 3.73$ x $10^5 P_w T^{-2}$ and T (temperature) is estimated. P_w (water vapor pressure) is deduced from N_w.

$(N_w)_E$

$(N_w)_G$

5) Now make new estimates of r_E and r_G from a suitable model atmosphere, with water vapor effect included. An isothermal model gives $\left[r_i\right]^2 = \left[r_i\right]^1 \mu_w \, (r_i)$.

$\left[r_E\right]^2$

$\left[r_G\right]^2$

6) Also use the water vapor information to gain an estimate of the refractivities at E and G. Here $N_E = N_D + (N_w)_E$.

$\left[N_E\right]^1$

$\left[N_G\right]^1$

7) From the values at hand for Φ_o and η_o $(= \mu_o r_o)$, set up estimates of $d\Phi_o / d\eta_o$ along $\overline{EG}$. This gradient is needed in retrieval if the method of Phinney and Anderson [1968, Eq. (46)] is used.

$\left. \dfrac{d\phi_o}{d\eta_o} \right|_{\overline{EG}}$

8) Employ inversion formula mentioned in Para. 7 above $r(\eta)\big|_{\overline{EG}}$

to find $r(\eta)\big|_{\overline{EG}}$.

9) From the relationship $\mu_i = \dfrac{\eta_i}{r_i} = \dfrac{\mu_i r_i}{r_i}$, obtain final N_E

estimates of N_E, N_G. N_G

10) Compute ρ_E, ρ_G from Dale and Gladstone's Law. ρ_E

ρ_G

2. Qualitative Summary of Errors

The errors inherent in this scheme appear to be rather damaging, especially in view of the present state of the art in securing the measured quantities. There is also a fundamental difficulty in the mathematical inversion process which is treated at length elsewhere in this report. Using the paragraph numbers listed above, the following kinds of errors are suspected:

STEP

1)   DEFG is not unilateral, the horizontal displacement being of the magnitude of a few tens of kilometers. Errors in satellite locations are reflected in errors in $\overline{r}_i$.

2,5)   The model atmosphere introduces an error.

3,6)   The purpose of the dual frequency measurement is to isolate the water vapor effect. The data would probably contain a major source of error.

4)   Estimation of T has to be done with some care. An iterative procedure based upon final, retrieved temperature is a possibility.

7,8)   The frequency of data point values in the vertical is another error source in the data inversion.

D. Differential Refraction for a Special Case

One of the factors contributing to the atmospheric extinction of radiation is the dispersive effect of differential refraction. This effect was studied by Roble [Fischbach, et al. 1965] for stellar radiation, and the wavelength dependence is slight. However, different satellite configurations may be expected to produce distinctive results in the extinction due to this cause.
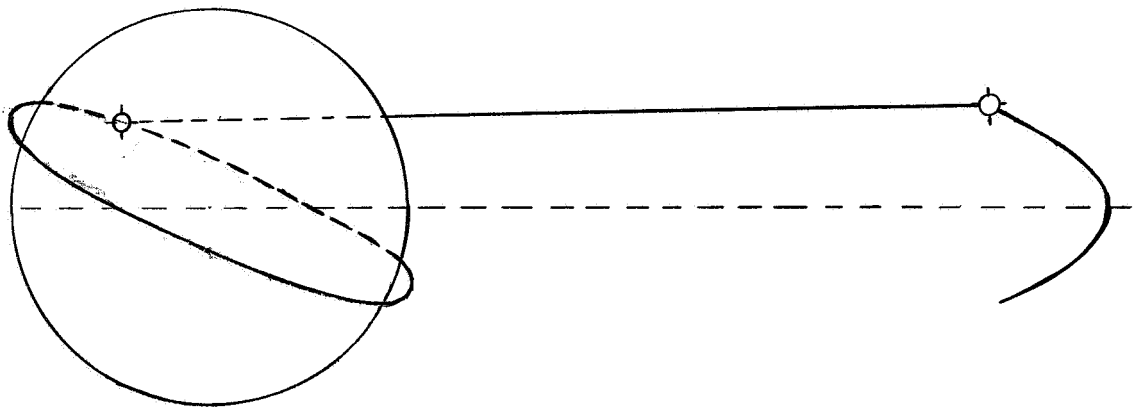

Fig. 6   Schematic drawing of dual-satellite system,
with one satellite synchronous.

Consider the orbiting pair in Fig. 6. The inner satellite has an altitude of 1100 km and the outer is synchronous at 35,700 km. An intensity reduction occurs when the microwave ray is dispersed, and this effect can be estimated when certain assumptions are made about the atmosphere. The previous study [Fischbach, et al. 1962] indicated that little difference is found between the outcome for an isothermal atmosphere and a standard model atmosphere.

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Hence the isothermal assumption will be applied here to a model suitable to the subtropics with the values for pressure (P), temperature (T), water vapor mixing ratio (W) given in Table 3. Tangent ray height (h_o) is another input in the table, and the formula for the isothermal case is

$$\frac{I}{I_o} = \left[1 + \frac{r_s \sin \theta}{H} \sqrt{\frac{2\pi r_o}{H}} \, (\mu_o - 1)\right]^{-1}$$

where

$$\theta = \frac{\pi}{2} + R_s - \sin^{-1}\left[\frac{r_o \mu_o}{r_s}\right]$$

r_s = orbital radius of satellite (km)

H = isothermal atmosphere scale height (km)

R_s = refraction angle of ray for the isothermal case

The resulting transmission factor is listed for a ray passing from the inner satellite to the outer one (ψ_1), and vice versa (ψ_2).

h_o	P	T	W	ψ_1	ψ_2
0 km	1014 mb	305°K	25 gm kg^{-1}	0.5%	4.3%
3	714	284	7	0.8	6.6
5	559	272	2	1.1	8.7
10	286	238	0	2.0	15.0
15	132	203	0	3.7	24.7
20	58	212	0	8.5	44.0
25	26	222	0	17.8	64.7

Table 3

In the case where radiation is transmitted from the inner orbiter through the Earth's atmosphere to the synchronous satellite, the extinction is severe,

with but 1.1% being transmitted at 5 km tangent height and 2.0% at 10 km.

IV. CONCLUSIONS AND RECOMMENDATIONS

A. Summary of Microwave Occultation Problems

The successful implementation of a multi-satellite microwave propagation system for acquiring Earth-atmosphere synoptic data faces many problems. Some have solutions that may be purchased (such as more satellites) and feasibility becomes an economic exercise. Other problems (aspherical stratification of water vapor) remain without suggested solution and feasibility becomes a question of tolerable accuracy. The following discussion is an attempt to list all of the known problems confronting a microwave system, with a brief discussion of their current status:

1. Inversion of a limited number of phase measurements (pseudo-occultation) to either refractivity or an atmospheric model.

The phase shifts can be interpolated into a continuous function and the Abelian inversion used. The impact parameter would be extrapolated and corrected at each level beginning at the top where $\mu = 1$. If water vapor is assumed absent for the moment, the retrieved function μ (r) would be transformed and integrated. These profiles would be compared with the hypothetical atmosphere to give probable errors. The atmospheric model, number of phase measurements, and phase measurement error (from any cause), can all be varied to yield a definitive error analysis.

Lusignan is pessimistic about the interpolation process and feels model-matching is required, which can utilize supplementary measurements such as surface temperature and climatological assumptions. The evaluation of the phase measurements and the analytic determination of error

are not possible with this method. An analysis cannot begin until the supplementary measurements are specified.

2. Refractivity resulting from density cannot be separated from refractivity caused by water vapor.

This problem exists below 300 mb where there is enough water vapor to cause trouble. Some correction is possible on a climatological basis. Some correction may be based on the signal strength fluctuation. Two frequencies operating in-an-out of the 1.35 cm band might eliminate this problem.

3. When refractivity is due to water vapor, it need not be approximately spherically stratified. The resulting error cannot be minimized by knowledge of the amount of water vapor on the path.

This problem has been examined by Lusignan, who hypothesized asphericity by a linear pressure gradient. We have analyzed this problem and found only even-powered derivatives to be of importance. We therefore believe Lusignan's result to be meaningless. We regard this problem to exist and be serious between 500 mb and the surface.

4. Relative velocity between spacecraft causes a phase shift which must be removed with great precision, or the positions known exactly.

Lusignan believes that the slave satellites can be moved "backwards" to the proper station and still be in the same orbit as the master. By a priori knowledge of the Earth's gravitation anomalies, the position of the slave can be continuously known with great precision relative to the master. He believes that a constant error will show as an ever-expanding atmosphere and can be completely removed without error. The amount of time required for this deduction and correction is not known. It must be done soon enough to use the data and redone as soon as a thruster is energized. Tatarskiy concludes that the position must be known to within 16 cm., which he regards as impossible.

5. Multi-path transmissions will occur when critical refraction is encountered. This is anticipated with normal strong water vapor gradients. The shortest such ray-path must be sorted out.

Lusignan states that improvements in equipment design can alleviate this problem. The weaker signals would be rejected.

6. Ionospheric contribution to the phase shift cannot be separated from the neutral atmospheric effect.

Proper choice of frequencies can alleviate this problem. No detailed analysis has been made. The Earth's ionosphere effect was corrected on the Mars experiment.

7. Equipment design:

Great frequency stability is required.

Stability and control are required for each satellite. Micro-thrusting is required on each slave satellite.. Satellite doppler reception, recording, and telemetry; data rates and accuracy requirements are severe. Data processing (on the ground) must be done in real time.

These requirements have been studied for some time by Lusignan. Approaches have been worked out for all but the last. A final technique has not been proposed; therefore no specific design can be evaluated. Since no data inversion method has been proposed, the real time processing cannot be evaluated.

B. Recommendations

The most pressing problem to the evaluation is the development of an analytic error technique so that any and all of the sources of error can be related to the retrieval of atmospheric parameters. This error analysis will not only be a deciding factor in the feasibility conclusions, but will also indicate the relative value of equipment design goals. Implicit in the error analysis is a method for data processing. The approach to be used is the application of equation (13) of Appendix A in a 360/50 program which parallels our present 7090 program for

equation (12).

It is recommended that the proponents of microwave systems make firm the specific embodiments they propose, so that realistic errors may be deduced.

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Appendix A

Abel Integral Inversion Technique

1. Abel Transform

If in a closed region a function of the two variables $f(x, y)$ has a continuous derivative with respect to x, the derivative of the integral of the function is the integral of the derivative; that is

$$\frac{d}{dx} \int_a^b f(x, y) \, dy = \int_a^b \frac{\partial f(x, y)}{\partial x} \, dy \qquad (1)$$

Thus if we consider a parameter in the limits , and write

$$F(x) = \int_{u(x)}^{v(x)} f(x, y) \, dy \qquad (2)$$

if $f_x(x, y) \equiv \dfrac{\partial f}{\partial x}$

then we may write

$$F(x) = \psi (u, v, x) \qquad (3)$$

and by the differentiation chain rule

$$F'(x) = \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial u} \frac{du}{dx} + \frac{\partial \psi}{\partial v} \frac{dv}{dx} \qquad (4)$$

therefore

$$F'(x) = \int_{u(x)}^{v(x)} \frac{\partial f}{\partial x} \, dy - u'(x) f(x, u(x)) + v'(x) f(x, v(x)) \qquad (5)$$

Now if n is a positive integer and $f(y)$ continuous in the closed interval $0 \leq y \leq x,$ consider the integral

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$$F_n(x) = \int_0^x \frac{(x-y)^n}{n!} f(y)\, dy \tag{6}$$

which will be of considerable interest.

Note particularly that

$$F_n'(x) = \int_0^x \frac{n(x-y)^{n-1}}{n\,(n-1)!} f(y)\, dy + 0 + 0 \tag{7}$$

which means $F_n'(x) = F_{n-1}(x)$ $\tag{8}$

similarly $F'_{n-1}(x) = F_{n-2}(x)$

and $F_1'(x) = F_o(x)$

where $F_o(x) = \int_o^x f(y)\, dy$ $\tag{9}$

but then $F_o'(x) = f(x)$ $\tag{10}$

so that $F_n^{(n+1)}(x) = f(x).$ $\tag{11}$

Thus we have shown that

$F_n(x)$ and its first n derivatives vanish when x = 0 and that the (n+1)-st derivative is f(x).

Therefore

$$\int_o^x \cdots \int_o^x f(x)\, dx^{(n+1)} = F_n(x) \tag{12}$$

And it follows that

$$F_n(x) = \int_0^x \cdots \int_0^x f(x)\, dx^{(n+1)} = \int_0^x \frac{(x-y)^n}{n!}\, f(y)\, dy. \tag{13}$$

If we define the function

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1}\, dx \text{ for } n > o \tag{14}$$

we can show that for positive integral n

$$\Gamma(n) = (n-1)!$$

Therefore we have

$$\int_0^x \frac{(x-y)^{n-1}}{(n-1)!}\, f(y)\, dy = \frac{1}{\Gamma(n)} \int_0^x (x-y)^{n-1} f(y)\, dy = G(x) \tag{15}$$

where $G(x)$ is actually the integration of $f(x)$ n times between o and x. If one defines the <u>differential operator</u> D and the <u>integral operator</u> D^{-1} where D^{-1} is $\int_0^x dx$ then we may conveniently say $G(x) = D^{-n} f(x)$. $\tag{16}$

$G(x)$ must of course vanish at $x = 0$, and must be differentiable n times with n-1 derivatives vanishing at $x = 0$ and the nth derivative equal to $f(x)$. Since $\Gamma(a) = (a-1)!$ for all $a > 0$ it is possible to <u>define a-th order integration</u> for all a where a is a positive real number:

$$D^{-a} f(x) \equiv \frac{1}{\Gamma(a)} \int_0^x (x-y)^{a-1} f(y)\, dy \tag{17}$$

We <u>define a-th order differentiation</u> as the inverse process, where if m is the

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integer $o < (m-a) < 1$

$$D^a f(x) = D^m D^{a-m} f(x) = (\frac{d}{dx})^m \frac{1}{\Gamma(m-a)} \int_o^x (x-y)^{m-a-1} f(y) dy \qquad (18)$$

From equations (1) and (14) applied to (17) and (18) we find that for all a:

$$D^{-a} D^a f(x) = f(x)$$

and

$$D^a D^b f(x) = D^b D^a f(x) \qquad (19)$$

for all a, b real.

Applying (19) to (18) we see

$$D^a f(x) = D^m D^{a-m} f(x) = D^{a-m} D^m f(x) \qquad (20)$$

thus

$$(\frac{d}{dx})^m \frac{1}{\Gamma(m-a)} \int_o^x (x-y)^{m-a-1} f(y) \, dy = \frac{1}{\Gamma(m-a)} \int_o^x (x-y)^{m-a-1} f^{(m)}(y) \, dy. \qquad (21)$$

Setting $a = -\frac{1}{2}$ and noting that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$:

$$D^{-1/2} f(x) = G(x) = \frac{1}{\sqrt{\pi}} \int_o^x (x-y)^{-1/2} f(y) \, dy \qquad (22)$$

may be inverted to:

$$D^{1/2} D^{-1/2} f(x) = D^{1/2} G(x) = \frac{1}{\sqrt{\pi}} \frac{d}{dx} \int_o^x (x-y)^{-1/2} G(y) \, dy =$$

$$\frac{1}{\sqrt{\pi}} \int_o^x (x-y)^{-1/2} G'(y) \, dy \qquad (23)$$

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providing that:

1) $G(x)$ is continuous in the closed integration interval

2) $G(0) = 0$

3) $\dfrac{d}{dx} \displaystyle\int_{0}^{x} (x-y)^{-1/2} G(y)\, dy$ be continuous over the closed interval

These restrictions obtain from the restrictions on equations (1), (6), and (7). The functions:

$$G(x) = \frac{1}{\sqrt{\pi}} \int_{0}^{x} (x-y)^{-1/2} f(y)\, dy \tag{22}$$

$$f(x) = \frac{1}{\sqrt{\pi}} \frac{d}{dx} \int_{0}^{x} (x-y)^{-1/2} G(y)\, dy = \frac{1}{\sqrt{\pi}} \int_{0}^{x} (x-y)^{-1/2} G'(y)\, dy \tag{23}$$

are known as the Abel transform.

2. Application

If we write the equation of a straight ray passing a planet in terms of planetocentric coordinates $r,\ \theta$: $r = r_{o} \sec \theta$
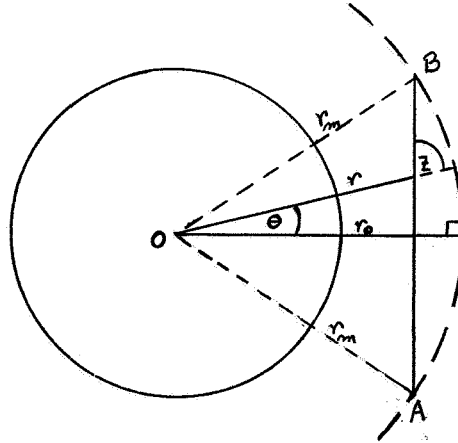

Fig. 7 Geometry of planetocentric coordinates;
$r = r_{o} \sec \theta$, $S = \overline{AB}$

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If we integrate along the path to obtain the distance out to some $r_{max} \equiv r_m$

$$s = (r^2 - r_o^2)^{1/2}$$

$$ds = (r^2 - r_o^2)^{-1/2} \, r dr$$

$$S = \int_A^B ds = 2 \int_{r_o}^{r_m} r (r^2 - r_o^2)^{-1/2} dr = 2(r_m^2 - r_o^2)^{1/2}$$

If we arbitrarily let r_m equal some radius beyond which the speed of radiation propagation is c, c being the speed of light in a vacuum, then all angular and phase perturbations will occur within this radius and we may assume straight and unperturbed propagation beyond.

Consider now that the planet has an atmosphere which may affect the speed of radiation propagation. If radiation of a given frequency enters a medium which slows the propagation, the wavelength will shorten as shown in Fig. 8.
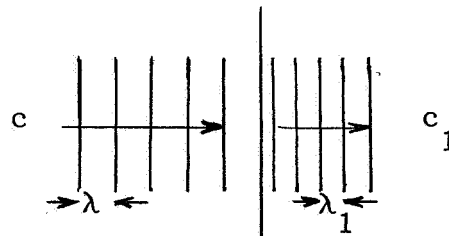

Fig. 8 Schematic diagram of the decrease in wavelength when propagation speed is reduced from c to c_1.

$$f = \frac{c}{\lambda} = \frac{c_1}{\lambda_1}$$

If it enters the second medium at a non-zero angle to the gradient, it must also be deflected as shown in Fig. 9. If z is the angle of the wavefront with the gradient, the angle of deflection will be $z_1 - z_2$
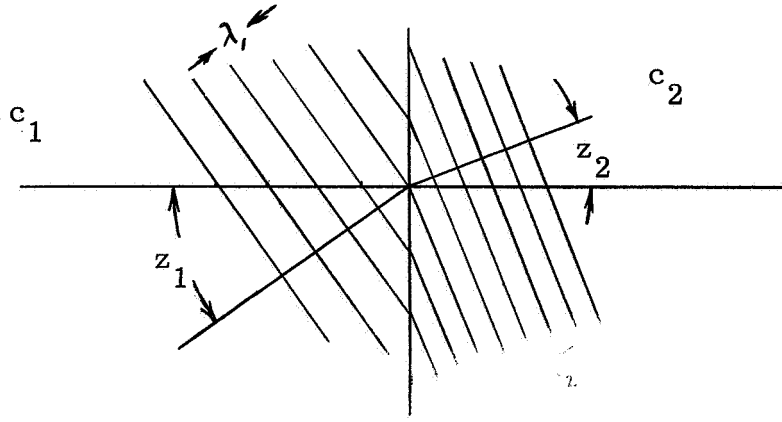
where

$$\frac{\sin z_2}{\sin z_1} = \frac{c_2}{c_1} \quad .$$

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Fig. 9 Schematic diagram of the deflection of a wavefront
when propagation speed is reduced from c_1 to c_2.

We will call $z_1 - z_2$ refraction, R, and define the ratio of propagation speed in
a vacuum to that in the medium as the index of refraction of the medium. If μ
is the index of refraction,

$$\mu_1 = \frac{c}{c_1}, \quad \mu_2 = \frac{c}{c_2}$$

then

$$\frac{\sin z_2}{\sin z_1} = \frac{\mu_1}{\mu_2} \tag{24}$$

If the medium has a continuously varying index of refraction, the ray will
curve and we can show that

$$\frac{dR}{ds} = \frac{1}{\mu} \nabla \mu \, \sin z \tag{25}$$

If $\nabla \mu$ lies along a planetary radius, r, and is the same along all radii

$$\frac{dR}{ds} = \frac{dR}{dr} \cos z \tag{26}$$

$$\frac{dR}{dr} = \frac{1}{\mu} \frac{d\mu}{dr} \tan z \tag{27}$$

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Noting central angle $\theta = z + R$ and $\tan z = -r \dfrac{d\theta}{dr}$,

$$\frac{dz}{\tan z} + \frac{dr}{r} + \frac{d\mu}{\mu} = 0 \tag{28}$$

follows immediately.

Thus:

$$\sin z \cdot r \cdot \mu = \text{constant} \tag{29}$$

The constant must be $\mu_o r_o$ at the point of tangency and also is the perpendicular distance to the ray asymptote. This constant is a very useful parameter, and serves as a label for the ray. It is the impact parameter discussed in Appendix B.

Since the total curvature of the ray is the refraction angle

$$R = \int_{-\infty}^{\infty} \frac{1}{\mu} \nabla\mu \, \sin z \, ds$$

We integrate

$$R = 2\int_{1}^{\mu_o} \frac{d\mu}{\mu} \tan z \tag{30}$$

to get

$$R = 2\mu_o r_o \int_{1}^{\mu_o} (\mu^2 r^2 - \mu_o^2 r_o^2)^{-1/2} \, d(\ln \mu) \tag{31}$$

or

$$R = 2\mu_o r_o \int_{\mu_o r_o}^{r_m} - (\mu^2 r^2 - \mu_o^2 r_o^2)^{-1/2} \frac{d(\ln \mu)}{d(\mu r)} \, d(\mu r) \tag{32}$$

Since

$$\frac{dR}{ds} = \frac{1}{\mu} \frac{d\mu}{dr} \sin z = \frac{1}{\mu} \frac{d\mu}{dr} \frac{\mu_o r_o}{\mu r} \ ,$$

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the path length from $(-\theta_m, r_m)$ to (θ_m, r_m) is:

$$S_R = 2 \int_{\mu_o r_o}^{r_m} \frac{\mu r \, d(\mu r)}{(\mu^2 r^2 - \mu_o^2 r_o^2)^{1/2}} \qquad (33)$$

If we consider the number of cycles of radiation along the path, Φ, it will be increased in inverse proportion to the ratio of propagation speed at each point. Thus if we refer to apparent path length, S_Φ, as the distance traversed by the same number of cycles in a vacuum,

$$S_\Phi = 2 \int_{\mu_o r_o}^{r_m} \frac{\mu^2 r \, d(\mu r)}{(\mu^2 r^2 - \mu_o^2 r_o^2)^{1/2}} \qquad (34)$$

The increase in apparent path length due to <u>retardation</u> along a refracted ray is

$$S_\Phi - S_R$$

It is most convenient to compare the apparent path length on the refracted ray to the path length on an unrefracted and unretarded ray, since the latter may be deduced geometrically and the former may be deduced from doppler data. In a real planetary atmosphere neither $\mu-1$ nor $\frac{d\mu}{dr}$ is zero; therefore rays are both refracted and retarded. There is no published scheme to separate refraction from retardation analytically when considering a single microwave frequency. Microwave schemes thus measure the total phase, usually by its time derivative, the doppler shift. Optical methods do not depend upon phase measurements but are susceptible to angular measurements so that retardation is neglected and refraction angle is the quantity measured. Since optical methods normally utilize a broad band of frequencies, the variations in ray paths cause a difficulty in measuring the refraction angle.

The change in apparent path length due to a real planetary atmosphere is the measurable microwave parameter

$$\frac{dS_\Phi}{dt} = \frac{d}{dt} \left[2 \int_{\mu_o r_o}^{r_m} \frac{\mu^2 r \, d(\mu r)}{(\mu^2 r^2 - \mu_o^2 r_o^2)^{1/2}} \right] \tag{34a}$$

which is integrated to give (11); and in optical measurements, the measurable parameter is the refraction angle

$$R = 2\mu_o r_o \int_{\mu_o r_o}^{r_m} \frac{d(\ln \mu)/d(\mu r)}{(\mu^2 r^2 - \mu_o^2 r_o^2)^{1/2}} \, d(\mu r) \tag{31}$$

It is important to note that both measurable quantities are functions of the impact parameter $\mu_o r_o$.

Applying the Abel transform developed above we can invert both equations to yield $\mu(\mu r)$, and implicitly $\mu(r)$:

$$\mu(\mu r) = \exp \left[-\frac{1}{\pi} \int_{\mu r}^{r_m} \frac{R(\mu_o r_o) \, d(\mu_o r_o)}{(\mu_o^2 r_o^2 - \mu^2 r^2)^{1/2}} \right] \tag{35}$$

and

$$\mu(\mu r) = \frac{1}{\pi} \frac{d(\mu r)}{dr} \int_{\mu r}^{r_m} \frac{d\Phi(\mu_o r_o)/d(\mu_o r_o) \, d(\mu_o r_o)}{(\mu_o^2 r_o^2 - \mu^2 r^2)^{1/2}} \tag{36}$$

Inputs of the measured parameter in either case must be continuous functions of the impact parameter, and $R(\mu_m r_m) = \dfrac{d\Phi(\mu_m r_m)}{d(\mu_m r_m)} = 0$ is likewise required for the transformation.

In the stellar-optical case, let subscript s denote position at a spacecraft

and let r_s and z (unrefracted) be known from spacecraft coordinates. Then μ_s = 1 and z_s = z (unrefracted)-R. Since $\mu_s r_s \sin z_s$ is the impact parameter, it is available immediately with each refraction angle measurement.

The situation in the microwave case is not the same; μ_s = 1, r_s and z (unrefracted) are known from spacecraft coordinates, but R is not measured nor available analytically. R can be approximated or obtained from an extrapolation, and iterated when necessary, downward from R = 0, datum point by datum point. Thus the lack of an analytic impact parameter does not preclude the use of an exact integral inversion when measuring phase path increase, but certainly complicates its application and increases the formidability of the data processing problem.

3. History of the Method

Abel solved the integral equation bearing his name in the early 19th century. The application of the equation to the inversion of the refraction integral was not known to Bessel, Sir James Ivory, Chauvenet, nor Lord Rayleigh, all of whom during the 19th century shared an interest in the refraction of an arbitrary atmosphere. The distinguished British mathematician, Harry Bateman, solved an identical problem in seismology by use of the Abel transform in 1909.

When the refraction project was begun in 1961 by L. M. Jones, we did not know the exact solution to the refraction integral. Various approximations were made without much satisfaction because analytic error estimates were impossible to perform. In January 1963 Bateman's solution lay unappreciated on my (Fischbach) desk. P. B. Hays, a doctoral candidate at the time, asked to consider the problem and quickly noted that the exact inversion had been found. We disseminated this solution immediately as a technical report to those we knew to be interested. Meanwhile new vistas had been opened and the error analyses based on the exact solution were completed, showing feasibility of the stellar refraction technique. The first announcement of the inversion and the error studies

were made before the annual meeting of the American Meteorological Society in January 1965 and later printed by invitation in their Bulletin. Following efforts included related studies showing that the Bateman solution was applicable to ozone absorption, scattering, and in fact all atmospheric perturbations of a continuous nature which were approximately spherically stratified. A summary of the exact inversion techniques was published by Hays and Roble in 1968.

On a closely related but separate tack, G. Fjeldbo was studying the problem of microwave occultation techniques for determining planetary ionospheres. Being also a refraction problem, the equations involved were grossly identical. In Fjeldbo's dissertation on the subject published in 1964 he has correctly written the exact expression for the ray path, but prior to demonstrating a solution, introduced the straight-ray approximations. The remainder of the dissertation then addresses itself to the solution of the approximation, including the Abel transform applied to the approximate ray. It was evident therefore that he did not know the inversion of the exact ray path equation. Subsequently, the success of the Mariner-Mars and-Venus probes gave Fjeldbo and his colleagues at Stanford and their co-investigators at Jet Propulsion Laboratory important and exciting data to process. However, the investigators steadfastly held to the straight ray approximation through 1967 and used the Abel transform to invert it. In 1968 Phinney and Anderson published a method on treating the Mars data with the exact inversion, developed a method for obtaining the impact parameter which we believe to be incorrect, and then proceeded to suggest approximations to simplify computations. Following this publication, Fjeldbo and Eshleman published the Mars data in final form using an Abelian inversion of the straight-ray and adding considerable justification for so doing, based on technological problems with the raw data. There the matter stands.

Another investigative team from MIT and the Raytheon Co. has proposed a Mars orbiter-pair to obtain ionospheric data and also a similar Earth technique. Their analysis is lengthy and comprehensive but they too apply the Abel transform

to straight rays in the same manner as Fjeldbo.

V. I. Tatarskiy of the USSR Academy of Sciences has published two recent papers on the application of both phase and refraction-angle measurements to the Earth's atmosphere. He, too, applies the Abel integral to the straight-ray approximation. He is aware of the exact solution because he has referenced our 1965 AMS Bulletin article. Some constraint may be due to the fact that his calculations were based on a two-satellite system which will not yield the impact parameter directly.

Appendix B
Impact Parameters

When considering the path of high frequency radiation through a refracting medium such as the atmosphere, an immediate consequence of Snell's law is the fact that for an individual ray the product of the index of refraction, the radius of curvature, and the sine of the zenith angle is a constant. This was shown in Appendix A, above. The constant thus may be thought of as a name or label for the ray.

In ray path analysis it is evident that this constant is of fundamental importance. It is not only the product $\mu_o r_o$ at the point of tangency and $r \sin z$ at any point beyond the atmosphere, but is also the radial perpendicular distance to the original undisturbed ray path, or "asymptote". It is, of course, the necessary parameter to trace a ray through a given atmosphere and as we have shown (Appendix A), the necessary parameter to infer an atmosphere from a given set of measurements.

It seems to us that the latter point has been sorely neglected by most analysts and experimenters. Except for stellar-refraction measurements, this constant is not analytically deducible immediately from the raw data. It can, however, be determined unambiguously to any desired accuracy and an exact analytic inversion performed. Other analysts have for the most part preferred to neglect the refraction of the ray or approximate it even while specifically engaged in determining the refractive index profile. This clearly is a hazardous procedure, but worse, unnecessary.

The fundamental parameter, $\mu r \sin z$, seemed to deserve a name, yet none appeared in the literature. "Ray-path-constant" was descriptive and "Snell's constant" was considered. In particle physics we noted that the perpendicular distance from the center of a force field to the velocity vector of an entering particle was called the "impact parameter". This seemed to correspond to the geometry of a ray entering the atmosphere. The analogy was a little crude since

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the atmospheric refractivity was not a central force field and the ray was not a particle, but we nevertheless termed the fundamental constant the "impact parameter".

This terminology was introduced by Fischbach, in January 1965 in a presentation at the annual meeting of the American Meteorological Society and reprinted in their Bulletin [Fischbach, 1965]. The exact inversion along the ray path was given at the same time. Later, the microwave experimenters used straight ray approximations and thus had little use for the fundamental parameter. However, in discussing microwave technique, Phinney and Anderson [1968] find an exact inversion for phase data providing they are functions of the "impact parameter". Interestingly, they do not reference our work. Also in 1968 the Soviet scientist Tatarskiy [1968 a, b] utilizes the term impact parameter, references our work, and means by it not the impact parameter but the radius at the tangent point, r_o! One can only speculate what part translations (both ways) played and whether Soviet nuclear physicists use the term or not. If not, Tatarskiy would have used the term because it is his variable of integration (as it was ours) and could not have realized the lack of analogy! The sequence of events proved amusing, if less than important.

We propose, therefore, that in ray path analysis the term "impact parameter" (of a ray) be used, and that it signify the perpendicular distance from a planetary center of gravity to the path the ray would have taken in a vacuum.